Classification of Diabetic Wounds Using Transfer Learning Model: EfficientNetB1 and ResNet50

George Morris William Tangka*[1], Andreas Rantepadang[2], Green Arther Sandag[3], Rolly Junius Lontaan[4], Wilsen Grivin Mokodaser[5]
[1]Sistem Informasi, Fakultas Ilmu Komputer, Universitas Klabat, Airmadidi
[2]Keperawatan, Fakultas Ilmu Keperawatan, Universitas Klabat, Airmadidi
[3, 5]Informatika, Fakultas Ilmu Komputer, Universitas Klabat, Airmadidi
[4]Teknologi Informasi, Fakultas Ilmu Komputer, Universitas Klabat, Airmadidi
e-mail: *[1]**gtangka@unklab.ac.id**, [2]rantepadang@unklab.ac.id, [3]greensandag@unklab.ac.id, [4]rolly.lontaan@unklab.ac.id, [5]wilsenm@unklab.ac.id

Abstract

Diabetic foot ulcers are a major complication of diabetes, and their early detection remains difficult in routine practice. We address this gap by evaluating transfer learning models—EfficientNet-B1 and ResNet-50—for automated diabetic-wound classification. Using a two-class image dataset ("Diabetic Wounds" vs. "Normal"), we fine-tuned both backbones with two optimizers (SGD, Adamax). Models were trained for 50 epochs (batch size 16) with standard data augmentation to improve generalization. Performance was evaluated by classification accuracy. EfficientNet-B1 with SGD achieved the best test accuracy (99.48%), outperforming EfficientNet-B1 with Adamax (98.86%), ResNet-50 with SGD (99.22%), and ResNet-50 with Adamax (97.66%). These results indicate that transfer learning—particularly EfficientNet-based architectures optimized with SGD—can provide highly accurate, automated screening of diabetic wounds. The approach shows promise for integration into clinical decision-support systems to assist timely triage and management, and motivates future work on multi-center, patient-level validation and evaluation across diverse skin tones and imaging conditions.

Keywords— Diabetic Wound, Transfer Learning, Deep Learning, ResNet50, EfficientNetB1.

1. INTRODUCTION

Diabetes Mellitus (DM) is a chronic metabolic disorder characterized by elevated blood glucose levels that progressively damage vital organs such as the heart, kidneys, eyes, and nerves [1]. One of its most critical complications is the diabetic foot ulcer (DFU), which often results in infection, hospitalization, and lower-limb amputation [2]. Rantepadang [3] observed that comorbid factors such as diabetes and hypertension substantially diminish the quality of life among patients with chronic illnesses, emphasizing the need for early detection and effective management strategies. Therefore, developing automated diagnostic systems for early diabetic-wound identification is crucial to improving treatment outcomes and reducing healthcare burdens.

Recent advancements in artificial intelligence (AI) and deep learning (DL) have transformed medical image analysis by enabling automatic feature extraction and pattern recognition from complex visual data [4]. Among these methods, Convolutional Neural Networks (CNNs) have demonstrated remarkable success in tasks such as lesion detection, image segmentation, and wound classification, often achieving diagnostic accuracy comparable to expert clinicians. However, CNNs require large annotated datasets and substantial computational resources, which are often unavailable in medical domains. Transfer learning provides an effective solution by reusing knowledge from models pre-trained on large datasets such as

ImageNet, allowing faster convergence and better generalization on smaller, domain-specific datasets [5]. In conclusion, by applying transferred knowledge, this method leads to a faster training phase and better overall results for a target task [6]. This approach is particularly beneficial in DFU classification, where acquiring large labeled data is challenging.

Previous research has demonstrated the success of transfer learning in DFU image analysis. Wang et al. [7] proposed a few-shot DFU classification framework based on a deep ResNet, trained on 146 clinical DFU images augmented to approximately 3,000 samples. Their model achieved 98.67% accuracy for three-class wound severity classification (zero, mild, and severe), demonstrating that deep residual learning is effective even with limited data. Liu, John, and Agu [8] employed EfficientNet for ischemia and infection classification on the DFUC2021 dataset, achieving 99% and 98% accuracies, respectively. Their results showed that the balanced scaling strategy of EfficientNet improves computational efficiency and outperforms conventional CNN ensembles in medical imaging. Ullah et al. [9] developed Eff-ReLU-Net, a variant of EfficientNet-B0, to classify multiple chronic wound types on Medetec and AZH datasets, obtaining 92.33% and 90.00% accuracies while demonstrating strong cross-dataset generalization. Debnath et al. [10] emphasized the sustainability aspect of AI by employing lightweight MobileNet-based models on the DFUC2020 dataset, achieving 97.8% accuracy suitable for real-time mobile deployment in low-resource clinical environments. These advancements build upon the foundational principles of EfficientNet, which introduced a compound scaling method that optimally balances network depth, width, and resolution to achieve superior accuracy with fewer parameters [11]. However, few studies have directly compared multiple CNN backbones and optimizers under a uniform dataset, split, and training protocol, leaving open the optimal configuration for DFU detection.

This study aims to evaluate and compare the performance of EfficientNetB1 and ResNet50 models for binary classification of diabetic wound and normal skin images using transfer learning. Both architectures are trained under standardized configurations to ensure a fair comparison, and two optimizers, Adamax and Stochastic Gradient Descent (SGD), are examined to assess their influence on training dynamics and classification accuracy. The proposed framework seeks to determine the optimal combination of CNN architecture and optimizer for efficient and reliable DFU detection. The novelty of this research lies in its integrative evaluation design that bridges architectural and optimization perspectives, contributing to the advancement of AI-based wound diagnostic systems capable of supporting clinical workflows efficiently and accurately.

2. RESEARCH METHODS

The methodology of this research follows a structured pipeline, as illustrated in Figure 1. The process begins with the foundational stage of data collection and preparation, where a specialized dataset of wound and normal skin images is curated, preprocessed to a uniform size, and augmented to create a more robust training set. The central phase of this study involves the implementation of a transfer learning strategy, where two prominent pre-trained CNN architectures, ResNet50 and EfficientNetB1, are adapted and trained for the specific classification task. To ensure a comprehensive comparison, each of these architectures is trained with two different optimizers, SGD and Adamax. The final stage is a thorough evaluation, where the classification performance of each model is quantitatively assessed on an independent test set to definitively identify the optimal combination of model architecture and optimizer for this clinical application.

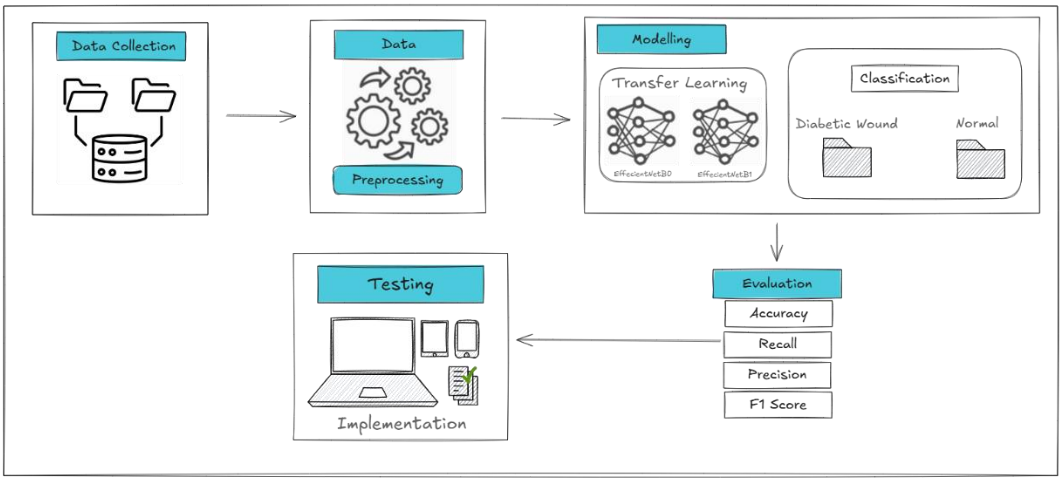

Figure 1. Research Design Methodology Framework

2.1. Data Collection

The dataset used in this study consists of 1,708 digital images collected from Kaggle with the title Collected and Categorized Wound Images Dataset and diabetic foot ulcer (DFU) [12] categorized into two classes: 'DiabeticWounds' and 'Normal'. To train and validate the models effectively, the dataset was split into a training set, a validation set, and a testing set. As shown in the data summary (Figure 2), 1,092 images were allocated for training, and 128 images were used for validation. A separate test set of 144 images was used for the final evaluation of the trained models [13][14].

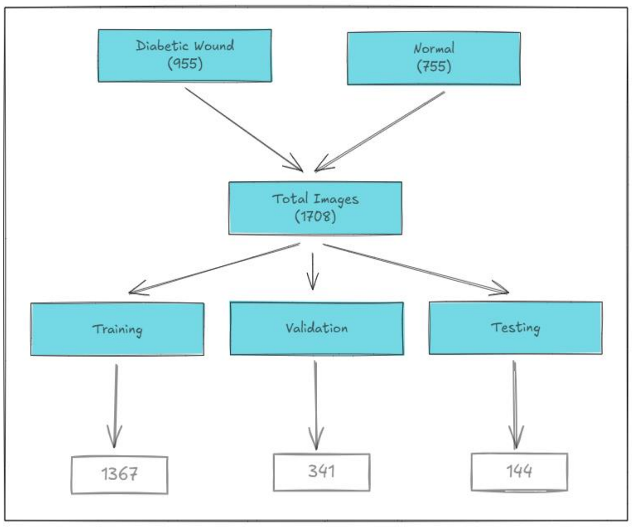

Figure 2. Total Dataset and Data Split Distribution

2.2. Data Preprocessing

To prepare the data for the modeling phase, all images underwent a critical preprocessing stage to ensure consistency and compatibility with the pre-trained architectures. Every image was resized to a uniform dimension of 224x224 pixels with 3 color channels (RGB), as this is the standard input size expected by the ResNet50 and EfficientNetB1 models or overall pre-trained CNN models [15]. This standardized dimension also decreases the time needed for the training phase [16]. To significantly enhance the model's ability to generalize to new, unseen images and to mitigate the risk of overfitting, a series of data augmentation techniques was applied in real-time to the training set. As illustrated in Figure 3, these transformations included random horizontal and vertical flips, random rotation, zooming, and contrast adjustments, which were integrated as the first layer of the model to create a more diverse and robust training experience.

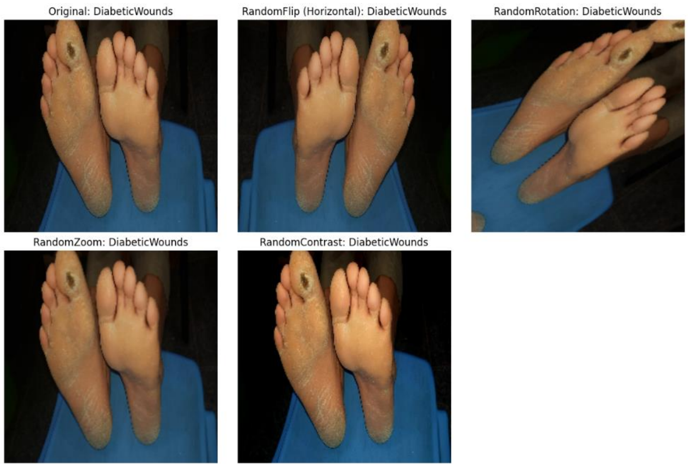

Figure 3. Example of Data Augmentation

2.3. Modeling

The modeling step, as in Figure 4, the deep learning process is conducted utilizing a transfer learning approach with the EfficientNet and ResNet50 models. The selection of these models is predicated on their state-of-the-art performance and widespread adoption in medical image analysis [17]. EfficientNet is distinguished by its compound scaling method, which systematically balances network depth, width, and resolution to achieve high accuracy with computational efficiency [18][19]. ResNet50, on the other hand, is renowned for its deep residual learning framework, which effectively mitigates the vanishing gradient problem in very deep networks, enabling the training of robust feature extractors [20][21].

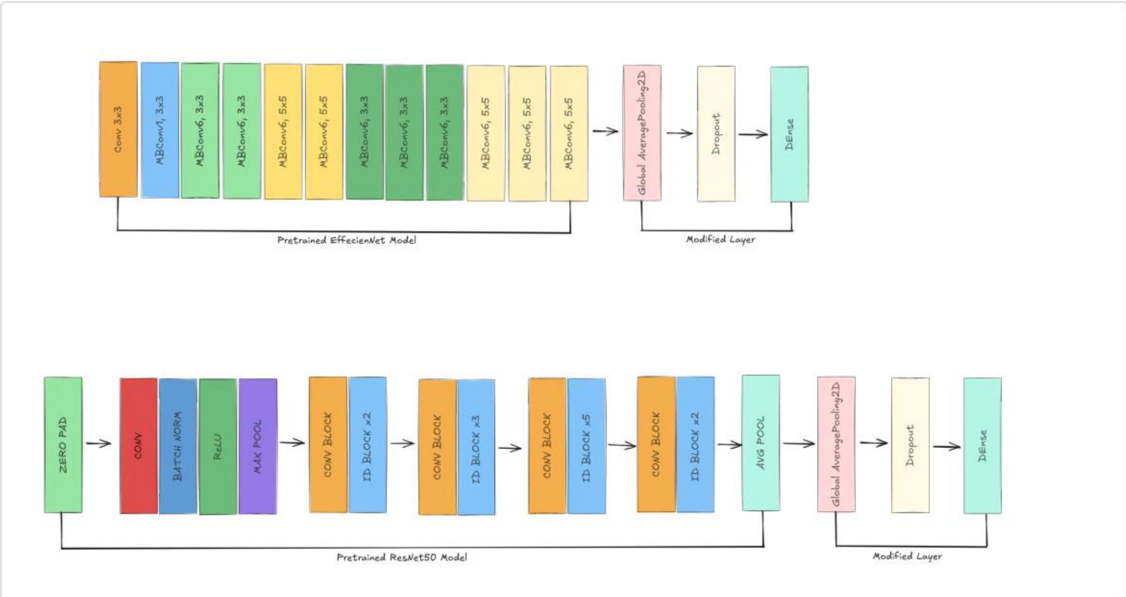

Figure 4. Modified Layer

As illustrated in the architectural diagrams, both models were adapted for this study's specific classification task. The pretrained convolutional bases of both networks comprising the MBConv blocks for EfficientNet and the convolutional and identity (ID) blocks for ResNet50 were preserved to act as powerful feature extractors. The original fully-connected top layers were then replaced with a custom classification head consisting of a Global Average Pooling2D layer to aggregate feature maps, a Dropout layer for regularization against overfitting, and a final Dense layer with a sigmoid activation function to do the final classification and produce the class probabilities. This customization allows the models to leverage rich, pre-learned features while tailoring the classifier to optimize performance for the specific dataset in this research.

In Table 1, the modeling phase of this study involved experimentation using transfer learning techniques with two prominent convolutional neural network (CNN) architectures: EfficientNetB1 and ResNet50. The experimental design was structured into two distinct scenarios to assess the impact of different optimization algorithms on model performance. Scenario 1 utilized the SGD optimizer, whereas Scenario 2 employed the Adamax optimizer. To ensure a controlled and fair comparison between the scenarios, all other critical hyperparameters were held constant. Specifically, all models were trained for 50 epochs with a low learning rate of 0.0001 to promote stable convergence during the fine-tuning process.

Table 1. Parameters

ScenarioID	Scenario	Optimizer	Epoch	Learning Rate	Loss Function	Batch Size
1	EffecienNetB1 ResNet50	SGD	50	0.0001	Binary	16
2	EffecientNetB1 ResNet50	Adamax	50	0.0001	Binary	16

A batch size of 16 was chosen, a common practice in medical image analysis to enhance generalization and manage memory constraints. Given the two-class nature of the problem, the Binary Crossentropy loss function was uniformly applied across all experiments. This structured methodology facilitates a direct evaluation of how the choice of optimizer interacts with each deep learning architecture in this specific classification task.

2.4. Evaluation

The primary objective of this phase was to precisely quantify the model's accuracy, precision, recall, and F1-score on the unseen test dataset. In the context of this medical diagnosis task, these metrics provide a multifaceted view of performance: accuracy gives an overall correctness score, precision is crucial for minimizing false positive diagnoses - FP (incorrectly identifying a wound), while recall is vital for minimizing false negatives (failing to detect an actual wound). Together, these metrics ensure a thorough and clinically relevant evaluation of each model's diagnostic capabilities, as defined by the following formulas. A true positive (TP) occurs when the model predicts DFU present, and the ground truth confirms DFU present; a true negative (TN) occurs when the model predicts DFU absent and the ground truth is DFU absent; a false positive (FP) occurs when the model predicts DFU present but the case is actually DFU absent (a false alarm); and a false negative (FN) occurs when the model predicts DFU absent but DFU is actually present (a missed case).

$$Accuracy = \frac{TP+TN}{TP+FP+FN+TN} \quad (1)$$

$$Precision = \frac{TP}{FP+TP} \quad (2)$$

$$Recall = \frac{TP}{TP+FN} \quad (3)$$

$$F1 - Score = \frac{2\ x\ Recall\ x\ Precision}{Recall+Precision} \quad (4)$$

3. RESULT AND DISCUSSION

3.1. Comparative analysis of the best transfer learning model

The comparative results for each architecture–optimizer pairing are summarized in Table 2. All configurations performed strongly (>97% accuracy), supporting the effectiveness of transfer learning for this task. Across metrics, EfficientNet-B1 consistently exceeded ResNet-50, and SGD outperformed Adamax for both backbones. The best result was EfficientNet-B1 + SGD with 99.48% accuracy, 1.0000 precision, 0.9897 recall, and 0.9948 F1. The next best, ResNet-50 + SGD, achieved 99.22% accuracy (precision 1.0000, recall 0.9792, F1 0.9895). EfficientNet-B1 + Adamax reached 98.96% accuracy (precision 0.9897, recall 0.9897, F1 0.9897), while ResNet-50 + Adamax obtained 97.66% accuracy (precision 0.9787, recall 0.9583, F1 0.9684). These head-to-head comparisons indicate that EfficientNet-B1 with SGD offers the most accurate and well-balanced performance for automated diabetic-wound classification.

Table 2. Comparison of All Models Used

Model	Optimizer	Loss Function	Accuracy	Precision	Recall	F1 Score	Scenario
EffecienNetB1	SGD	Binary	99.48%	1.0000	0.9897	0.9948	1
ResNet50	SGD	Binary	99.22%	1.0000	0.9792	0.9895	1
EffecienNetB1	Adamax	Binary	98.96%	0.9897	0.9897	0.9897	2
ResNet50	Adamax	Binary	97.66%	0.9787	0.9583	0.9684	2

Figure 5 illustrates the model's training and validation performance across 50 epochs for EfficientNetB1 with SGD. The loss graph on the left shows a sharp decrease in value for both training and validation data during the initial epochs, which then continues to decrease slowly until reaching a very low value. Epoch 50 is marked as the best epoch for loss. Meanwhile, the accuracy graph on the right shows a very rapid and significant increase, where the training and validation accuracy reach a very high level (approaching 1.0) and remain stable. The best epoch for accuracy is marked at epoch 23.

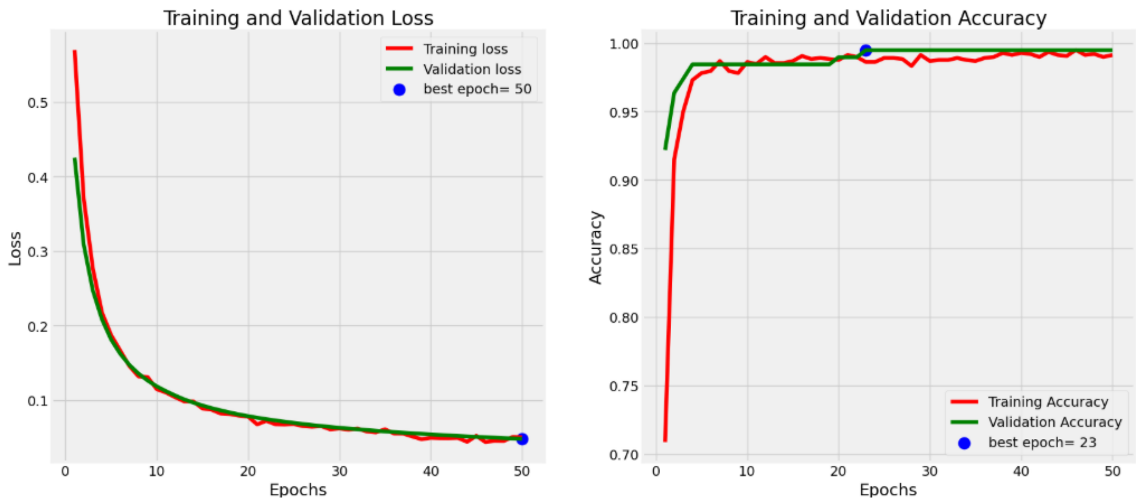

Figure 5. EfficientNetB5 with SGD Loss and Accuracy Graph

Figure 6 illustrates the model's training and validation metrics for loss and accuracy over a 50-epoch run for ResNet50. The model demonstrates strong performance, characterized by a steady decrease in loss (left graph), reaching its optimum at epoch 45, and a rapid surge in accuracy (right graph), which quickly plateaus at a very high level, with the best epoch for accuracy occurring at epoch 18. The minimal gap and close tracking between the training and validation curves across both plots confirm that the model learns effectively and generalizes well, showing no significant signs of overfitting.

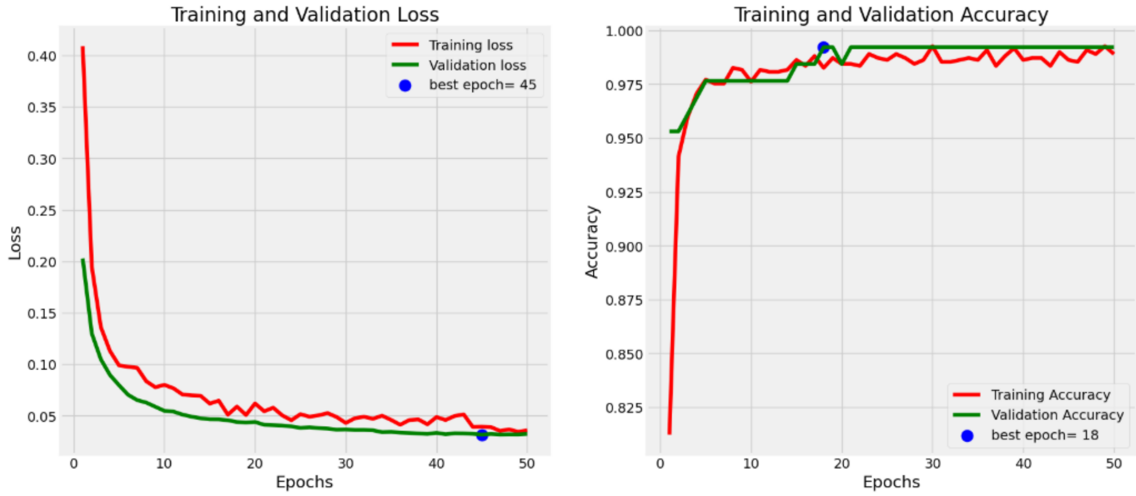

Figure 6. ResNet50 with SGD Loss and Accuracy Graph

Figure 7 demonstrates that the model EfficientNetB1 with SGD performs exceptionally well in classifying the DiabeticWounds class (True Positives: 95), with minimal misclassifications. The Normal class also shows extremely high performance, with 96 correct classifications.

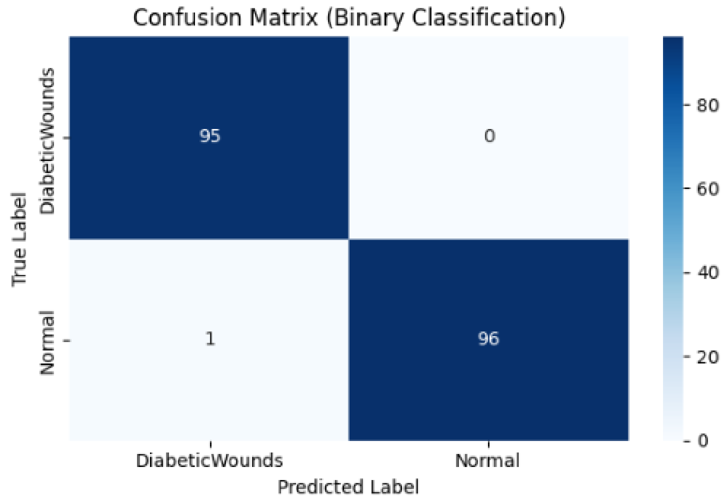

Figure 7. EfficientNetB5 with SGD Confusion Matrix

The only misclassification observed is a single instance where a Normal case was incorrectly classified as DiabeticWounds (False Positive: 1). Notably, the model achieved a perfect score in identifying all actual DiabeticWounds cases, with zero instances being misclassified as Normal (False Negatives: 0). This single error is minor and indicates the model has a very strong capability to differentiate between the two classes, demonstrating a near-perfect classification performance on this dataset.

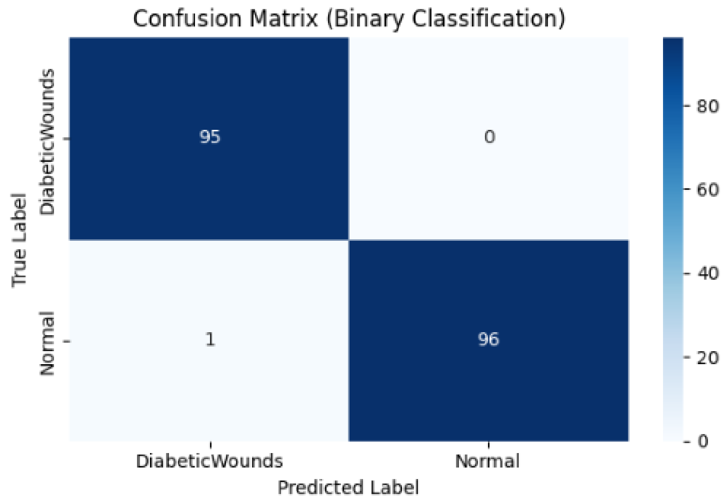

Figure 8. with SGD Confusion Matrix

Figure 8 demonstrates that the model ResNet50 with SGD performs exceptionally well in classifying the DiabeticWounds class (True Positives: 80), with minimal misclassifications. The Normal class also shows extremely high performance, with 47 correct classifications. The only misclassification observed is a single instance where a Normal case was incorrectly classified as DiabeticWounds (False Positive: 1). The model also achieved a perfect score in identifying all actual DiabeticWounds cases, with zero instances being misclassified as Normal (False Negatives: 0). This single, minor error confirms the model's robust capability to differentiate between the two classes, demonstrating a near-perfect classification performance on this dataset.

3.2. Comparison with Related Research

Table 3 compares the results of this research with several previous related studies in the field of wound image classification, particularly focusing on diabetic foot ulcers (DFU). This study is contextualized against notable prior research conducted by Wang et al. [7], Liu et al. [8], Ullah et al. [9], and Debnath et al. [10] to provide a performance benchmark. In this research, by utilizing the EfficientNetB1 and ResNet50 models on a custom-collected and categorized wound image dataset, a superior accuracy of 99.48% was achieved. This result demonstrates that the proposed approach is not only highly accurate but also significantly competitive when compared to previous works.

Table 3. Comparison with Other Related Research

Research Name	**Best Model**	**Dataset**	**Accuracy**
Wang *et al.*	ResNet50	DFU-classification	98.67%
Liu *et al.*	EfficientNetB1	The Diabetic Foot Ulcers Grand Challenge (DFUC) 2021 dataset	99.46%
Ullah *et al.*	Eff-ReLU-Net	Medetec Wound Datasets	92.33%
Debnath *et al.*	DenseNet and MobileNet	The Diabetic Foot Ulcers Grand Challenge (DFUC2020)	97.8%
This Research	**EfficientNetB1**	**Collected and Categorized Wound Images Dataset and diabetic foot ulcer (DFU)**	**99.48%**

Our study primarily focuses on developing a highly accurate classification model for wound images, as this is crucial for the early diagnosis and management of diabetic foot ulcers. Although each study may employ different datasets and methodologies, the inclusion of related research in this comparison is intended to highlight the effectiveness of our chosen deep learning architectures within this specific medical domain. Furthermore, by showcasing the superior performance of our models in achieving 99.48% accuracy on a custom dataset, we aim to emphasize their ability to handle real-world image variations while attaining state-of-the-art accuracy.

3.3. Testing or Implementation of Best Model

In Figure 9, the practical implementation of the best model, EfficientNetB1 with SGD, was tested to demonstrate its classification capabilities on a single, previously unseen image. As shown, an image of a diabetic foot wound was used as input, the model correctly classified it as 'DiabeticWounds' with an overwhelmingly high confidence score of 0.9941 (99.41%), while assigning a negligible probability of only 0.0059 to the 'Normal' class. This successful, high-confidence prediction on a novel data point serves as a practical validation of the model's performance, confirming that EfficientNetB1 is not only effective based on statistical metrics but is also robust and reliable for real-world applications.

Figure 9. Model Classification Testing

4. CONCLUSION

This study developed and evaluated transfer-learning models for diabetic-wound classification and found that EfficientNet-B1 with SGD achieved the highest accuracy (99.48%). This superior performance likely stems from EfficientNet’s compound scaling (balanced depth/width/resolution that improves feature efficiency) and SGD’s stable convergence and regularization-like effect, which together enhance generalization on a moderately sized dataset. Practically, the model offers a strong baseline for automated DFU screening and integration into clinical decision-support to support earlier triage and management. Future work should include multi-center, patient-level prospective validation, robustness testing across diverse skin tones and imaging conditions, and calibration/uncertainty assessment to ensure safe deployment. The comparative framework introduced here— a controlled, head-to-head evaluation of CNN backbones and optimizers under identical data splits and training protocols—helps identify optimal architecture–optimizer pairings for medical image classification. This protocolized approach is reproducible and transferable beyond DFU to related clinical imaging tasks.

For future work, the robustness of the model could be further enhanced by expanding the dataset to include a wider diversity of images, such as those from different ethnic groups, lighting conditions, and encompassing various stages of wound severity. Additionally, the model's utility could be extended by transitioning from binary classification to a multi-class problem, such as identifying different types of wounds or differentiating between infection and ischemia. To increase clinical trust and acceptance, implementing explainability techniques like Grad-CAM would be crucial to provide visual insights into the model's decision-making process. Furthermore, the deployment of the optimized model into a user-friendly mobile or web-based application is a logical next step to maximize its accessibility and practical utility in real-world clinical settings. Finally, exploring more recent architectures, such as Vision Transformers (ViT), could potentially yield further improvements in classification performance and efficiency.

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